Applied Spectral Fractal Dimension

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Outline

- Introduction
- The fractal dimension
- Measuring fractal dimension (FD)
- Spectral fractal dimension (SFD)
- Algorithm of the SFD
- Practical Application of SFD
- Conclusions

Introduction

reat expectations in 1980s in connection h practical applications of fractals

- simulation of chaotic phenomena (earthquake, tornado)
- examination of material (solid-state) structure
- modelling real processes
- computer animation, etc.

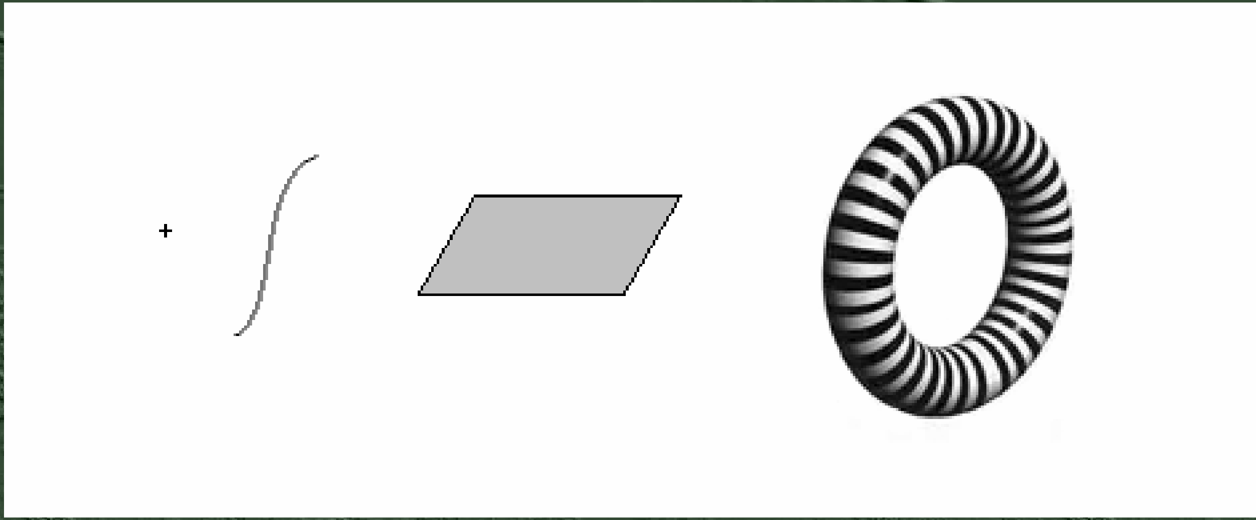
hat does at mean the fractal?

- Self-similar
- Generated
- Natural
- Beautiful
- Simple, etc.

Classical ideas on dimension

ell-known:

- 0, 1, 2, 3, … - integer
- independent
- simple (mathematical description and apply)



The fractal dimension

ndamental findings:

1. man-made objects are well defined in Euclidean geometry
2. natural objects can often best be modelled by fractal geometry

story:

- Niels Fabian Helge von Koch /1904/ - Koch Island /Snowflake or Koch Curves/
- Felix Hausdorff /1919/ - defined a dimension for point-sets /fraction/
- Benoit B. Mandelbrot /1975/ - term fractal was coined
- Benoit B. Mandelbrot /1983/ - Hausdorff-Besicovitch dimension is greater than the topological dimension
- Richard F. Voss /1985/ - popular algorithms for computing

Theoretical determination of the fractal dimension

t (X,d) be a complete metric space. Let $A \in H(X$
t $N(\varepsilon)$ denote the minimum number of balls
lius ε needed to cover A. If

$$FD = \lim_{\varepsilon \to 0} \left\{ Sup \left\{ \frac{Ln N(\bar{\varepsilon})}{Ln(1/\bar{\varepsilon})} : \bar{\varepsilon} \in (0, \varepsilon) \right\} \right\}$$

sts, then FD is called the fractal dimension of A.

Methods of Computing Fractal Dimensions

- east Squares Approximation /theoretical/
- /alking-Divider /practical to length/
- ox Counting /most popular/
- rism Counting /for a one dimensional signals/
- psilon-Blanket /to curve/
- erimeter-Area relationship /to classify diff. types images
- ractional Brownian Motion /similar box counting/
- ower Spectrum /digital fractal signals/
- ybrid Methods /calculate the fractal dim. of 2D using 1D methods/
- thers:
  - Correlation Dimension
  - Information (Rényi) Dimension
  - Lyapunov Dimension

omputing structural fractal dimension

3ox Counting

• most popular algorithms
• for computing the fractal dimensions of signals and images

$$FD = \frac{\log \frac{L_2}{L_1}}{\log \frac{S_1}{S_2}}$$

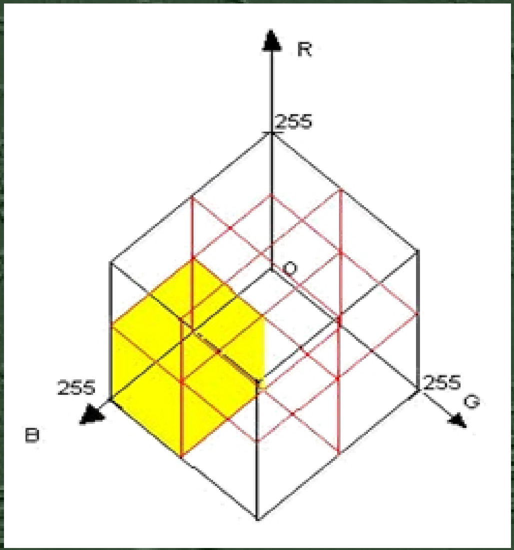
ere

- L1 and L2 measured length on curve,
- S1 and S2 metrics (resolution on image or signals)

Measuring structural fractal dimension (FD)

eature of images /digital images/

- Size – 1024x768 pixels
- Colour depth – 3x8=24 bits



Segmentation of image

Take four boxes based on original image / halving from the sides/

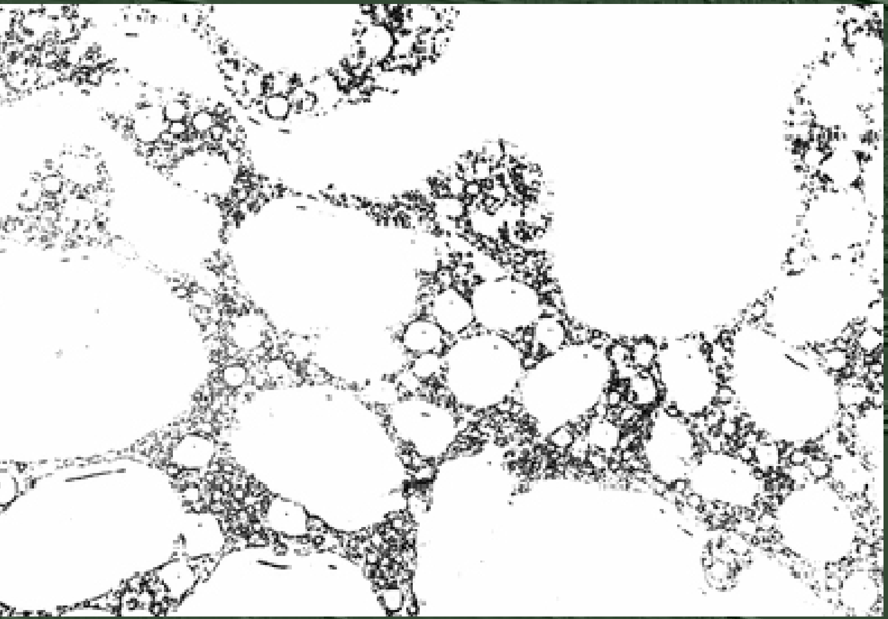
Which box content valuable pixel(s)

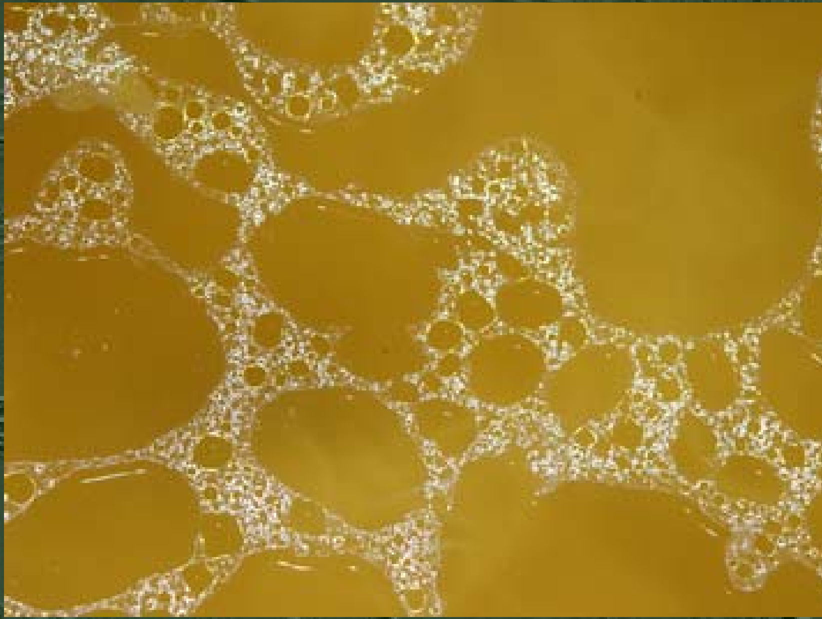
Save the number of valuable boxes

Repeat 2. - 4. paragraphs, than each of side will one pixels

Calculate dimension a form

Extension of fractal dimension





FD = 1,99	FD = 1,99

Spectral fractal dimension (SFD)

Let spectral fractal dimension (SFD) be:

$$SFD = \frac{\log \frac{L_{S2}}{L_{S1}}}{\log \frac{S_{S1}}{S_{S2}}}$$

here L_{S1} and L_{S2} are measured spectral length on N
mension colour space, S_{S1} and S_{S2} are spectral metric
pectral resolution of the image).

SFD in practice

{1, 3, 4, 6, 32, 60, 79, 126,…} colour space dimensio
r bands where

- N=1 black and white or greyscale image,
- N=3 RGB, YCC, HSB, IHS colour space image,
- N=4 traditional colour printer CMYK space image
- N=6 photo printer CCpMMpYK space image or Landsat ETM satellite image
- N=32 DAIS7915 VIS_NIR or DAIS7915 SWIP-2 sensors
- N=60 COIS VNIR sensor
- N=79 DAIS7915 all
- N=126 HyMap sensor

Typical spectral resolution:

- Threshold image -1 bit
- Greyscale image - 2-16 bits
- Colour image - 8-16 bits/bands

Computing SFD

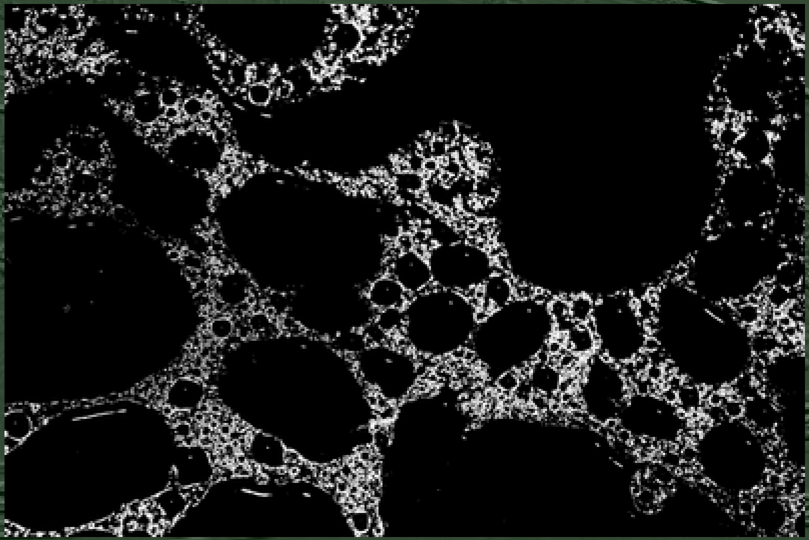
$$SFD_{measured} = \frac{n \times \sum_{j=1}^{n-1} \frac{\log(BM_j)}{\log(BT_j)}}{n-1}$$

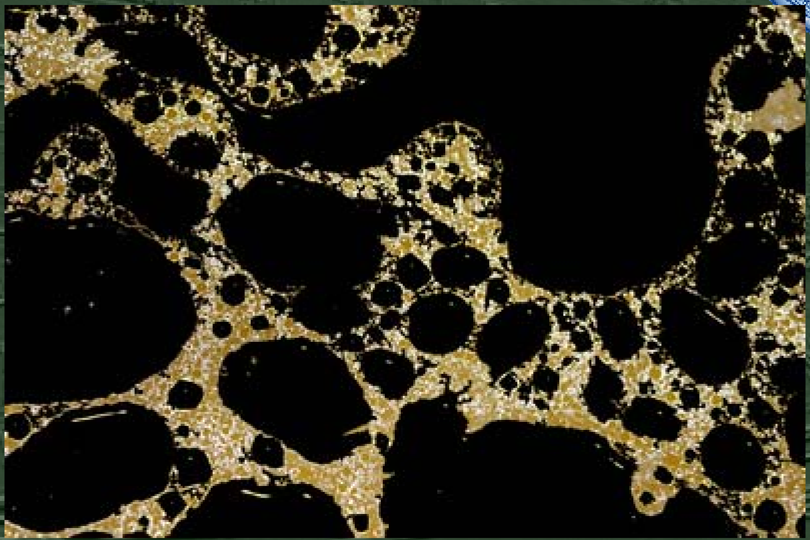
ere

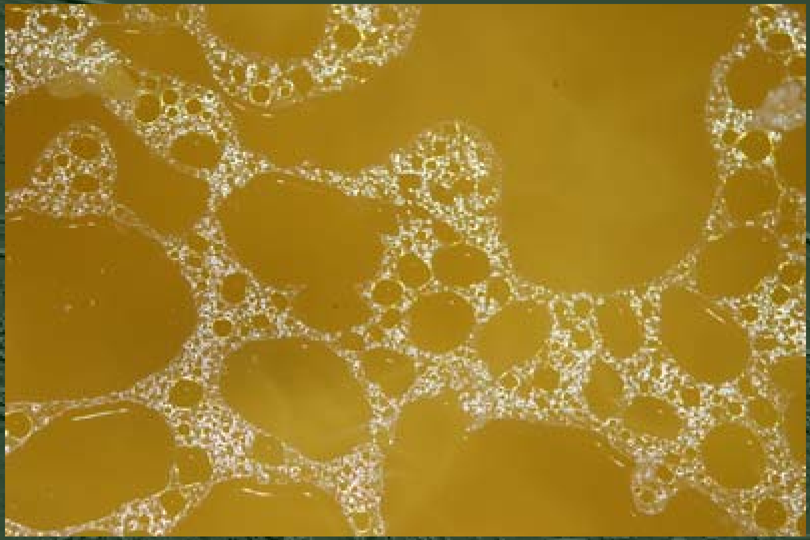
BM_j - number of spectral boxes containing valuable pixels in case of j-

BT_j – total number of possible spectral boxes in case of j-bits

Comparison FD and SFD





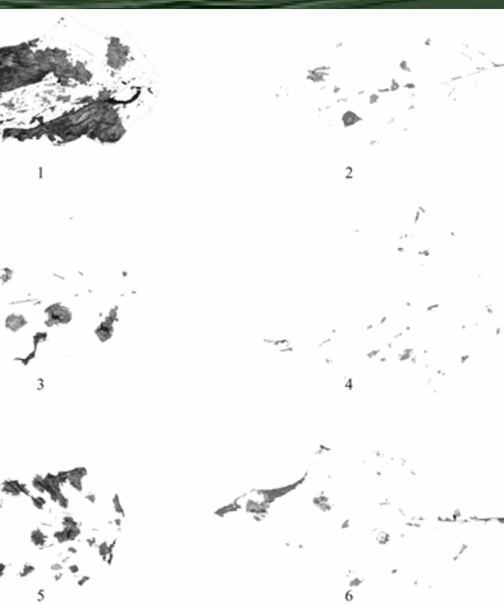


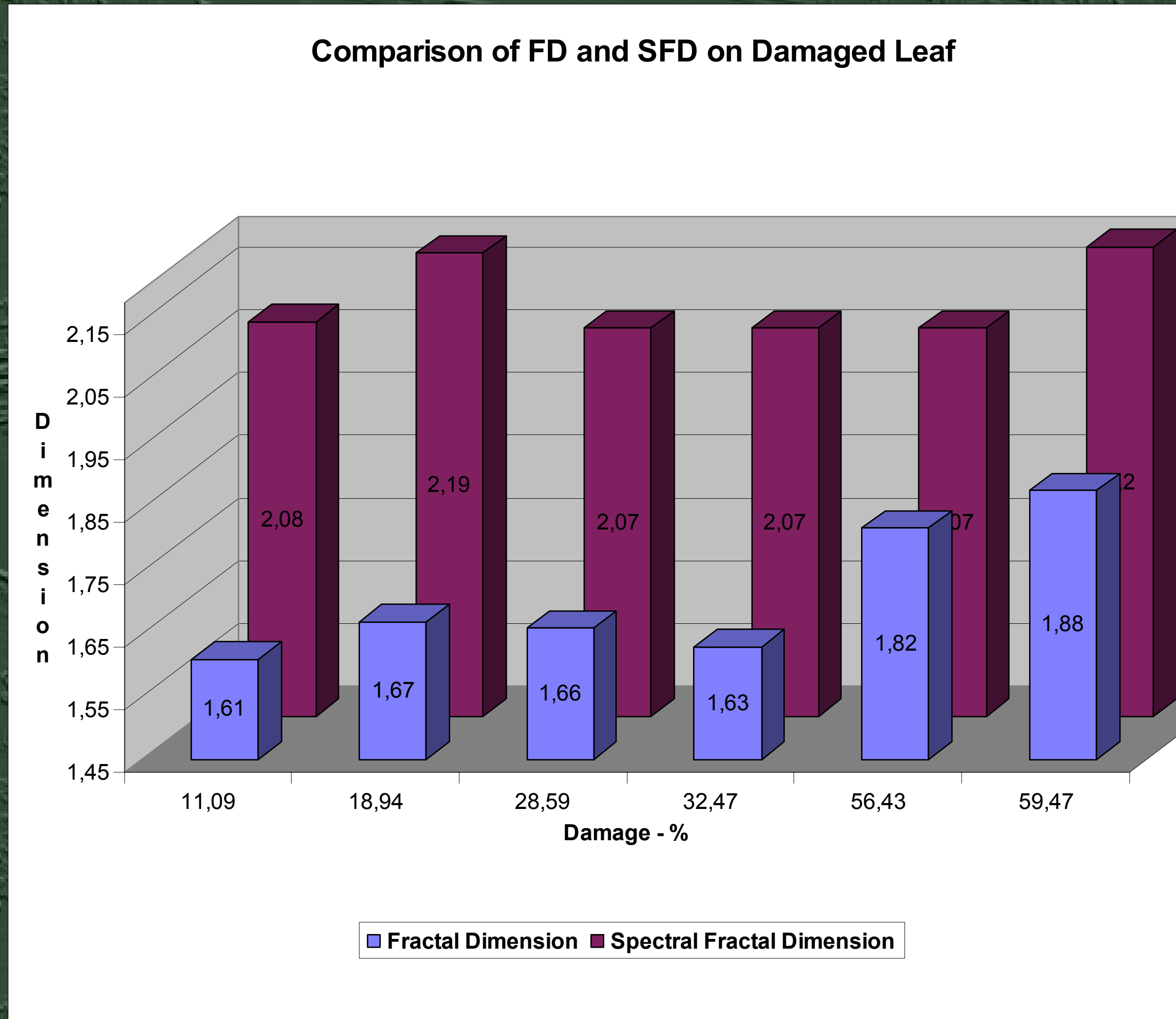
FD = 1,99 SFD = 1,14	FD = 1,99 SFD = 2,49
FD = 1,99 SFD = 2,51	

FD and SFD on Psychovisual Comparison of Image mpressing Methods under Laboratory Circumstances

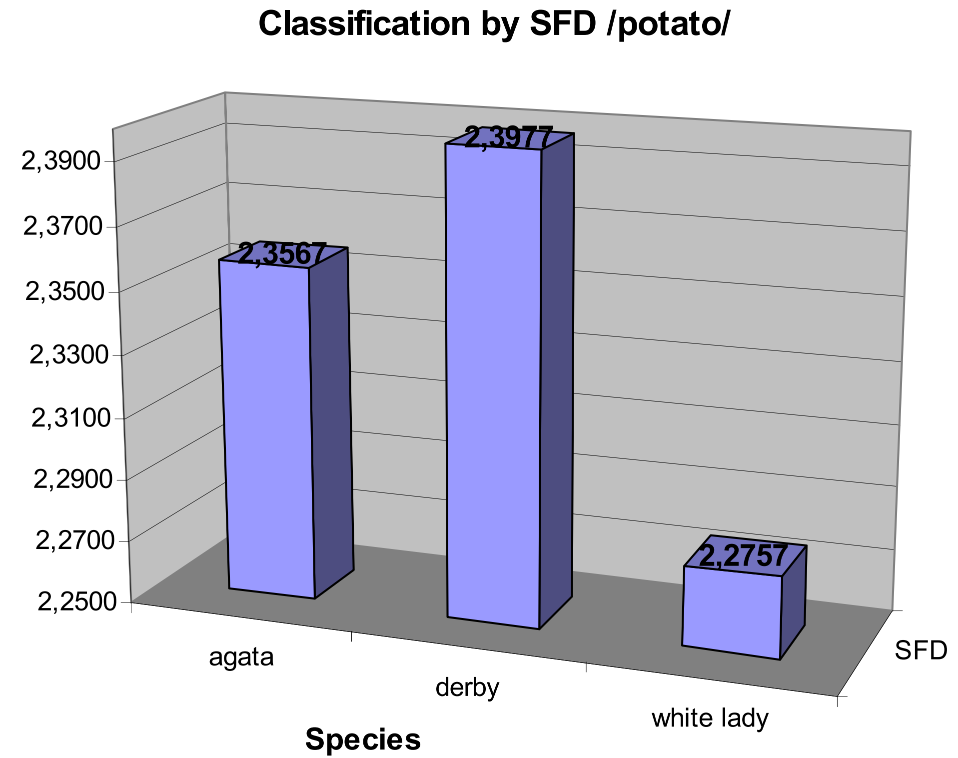
SFD = 2,31	SFD = 2,68
SFD = 2,56	

Damage of leaf



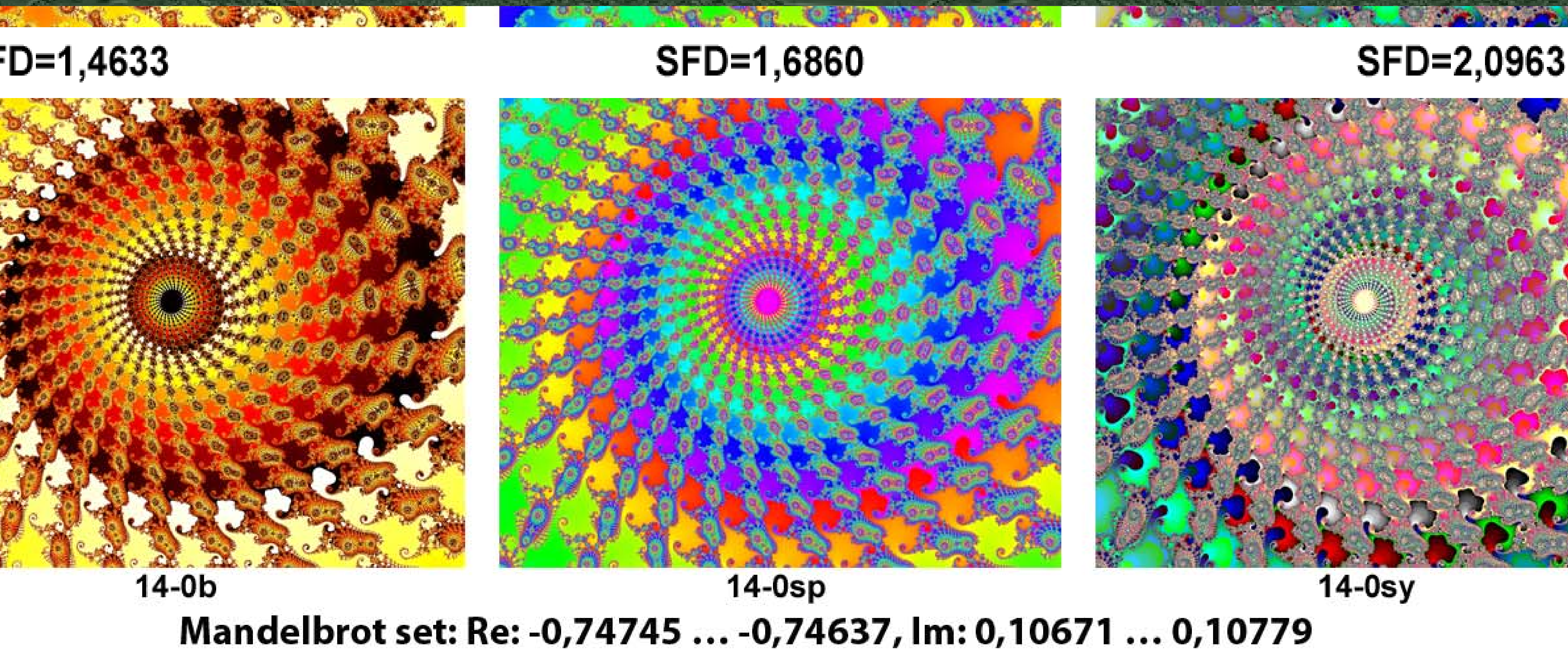


SFD to Image Classification



	agata	derby	white lady
SFD	2,3567	2,3977	2,2757

SFD on Fractals



Conclusions

- Useful information on structure as well as shades
- SFD can perfectly be used to characterize (multi-, yper spectral) images
- SFD and FD are significant parameters in the lassification
- SFD can be an important and easily measurable arameter of natural processes
- The applied SFD method give practically applicable esults in case of optional number of dimension

More info: www.georgikon.hu/digkep/sfd/index.htm